

Integration by substitution

1. (a) $\frac{2^x e^{ax}}{a + \ln 2} + C$ (b) $\frac{1}{2} \left(\frac{4^x}{\ln 2} + \frac{9^x}{\ln 3} + \frac{2^{2+x} 3^x}{\ln 6} \right) + C$ (c) $\ln[\ln[\ln[x]]] + C$ (d) $\tan^{-1} e^x + C$
 (e) $\frac{\ln(b + ae^x)}{a} + C$ (f) $2\sqrt{1+e^x} + C$ (g) $2e^{\sqrt{x}} - 6\sqrt{x} + C$ (h) $\frac{1}{2} \ln^2(2x) + C$
 (i) $e^{\sin x} + C$ (j) $-\frac{(1-e^x)^{1+n}}{1+n} + C$ (k) $-\frac{1}{1+\ln|x|} + C$ (l) $\frac{1-\ln 2}{4}$
 (m) $\frac{1}{8}(4-3\ln 3)$ (n) $\frac{1}{2\ln(3/2)} \ln \left| \frac{3^x - 2^x}{3^x + 2^x} \right| + C$ (o) $\frac{1}{3} \sec 3x + C$ (p) $\frac{1}{3} \tan(3x+7) + C$
 (q) $-\operatorname{Csc} x + C$ (r) $-\frac{\cos 2x}{2} + \frac{\cos 4x}{16} - \frac{\cos 8x}{32} + C$ (s) $-2\ln\left(\cos \frac{x}{2}\right) + C$
 (t) $\frac{1}{2} \ln(1+2\sin x) + C$ (u) $-\frac{1}{2} \cos(x^2) + C$ (v) $\frac{\tan^3 x}{3} + C$ (w) $\frac{5x}{2} - \cos 2x - \frac{3}{4} \sin 2x + C$
 (x) $-\frac{1}{2} \cot(x^2) + C$ (y) $-\cot x + \tan x + C$ (z) $\frac{2\sqrt{a+b\sin x}}{b} + C$
2. (a) $-2\cos\sqrt{x} + C$ (b) $\frac{\sin 2x}{4} - \frac{\sin 4x}{8} + C$ (c) $-\frac{\cos x}{2} - \frac{\cos 3x}{6} + C$
 (d) $-\ln(\cos\sqrt{1+x^2}) + C$ (e) $\frac{1}{AB} \tan^{-1}\left(\frac{A}{B} \tan x\right) + C$ (f) $\ln|\tan x| + C$
 (g) $-\cot\left(x + \frac{\pi}{4}\right) + C$ (h) $\frac{1}{2} \tan^{-1}(\sin^2 x) + C$ (i) $\frac{3}{2}(\sin x - \cos x)^{2/3} + C$
 (j) $\frac{1}{2\sqrt{2}} \ln\left(\frac{-1+\sqrt{2x-x^2}}{1+\sqrt{2x-x^2}}\right) + C$ (k) $2a\left(\tan^{-1}\sqrt{\frac{a+x}{a-x}} - \frac{\sqrt{a^2-x^2}}{2a}\right) + C$ (l) 0 (m) $\frac{2}{375}(3\sqrt{3}-7)$
3. $\frac{1}{3}(2a^2+x^2)\sqrt{x^2-a^2} + C$
4. (a) $\frac{\ln|x^2+4x+3|}{2} - \ln|2+x| + C$ (b) $\frac{1}{\sqrt{3}} \tan^{-1} \frac{2x-1}{\sqrt{3}} + \frac{1}{3} \ln|1+x| - \frac{1}{6} \ln|x^2-x+1| + C$
 (c) $\ln(x-2) - \ln(x-1) + C$ (d) $\frac{1}{2(b^2-x^2)} + C$
 (e) $\frac{1}{78}[(4\sqrt{3}-3)\ln|\sqrt{3}-x| + 6\ln|4+x| - (4\sqrt{3}+3)\ln|\sqrt{3}+x|] + C$ (f) $\frac{-1+99x-4851x^2}{470547(x-1)^{99}}$
 (g) $\frac{1}{3}(x+1)^{3/2} - \frac{1}{3}(x-1)^{3/2}$ (h) $\frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)}$ (i) $-\cos x - \frac{1}{2} \cot x \csc x + \frac{3}{2} \ln\left|\cot \frac{x}{2}\right| + C$
 (j) $-\frac{\csc^2 x}{2} + 3\ln|\tan x| + \sec^2 x + \frac{\sec^4 x}{4} + C$ (k) $\ln\left|\tan \frac{x}{2}\right| + \sec x + \frac{\sec^3 x}{3} + C$ (l) $-\frac{\cos 4x}{8} - \frac{\cos 6x}{12} + C$
 (m) $2\sqrt{ax+b}\left(\frac{9x^2}{2} - 2x\right) + C$ (n) $\frac{\sqrt{6x-9x^2}}{\sqrt{x}}(6x^{5/2} - 4x^{3/2})$ (o) $\frac{x^2}{2} - \frac{x}{2}\sqrt{x^2-1} + \frac{1}{2} \ln|x+\sqrt{x^2-1}| + C$
 (p) $\frac{3}{2} \cos \frac{x}{6} - \frac{3}{10} \cos \frac{5x}{6} - \frac{3}{14} \cos \frac{7x}{6} + \frac{3}{22} \cos \frac{11x}{6}$ (q) $\csc(a-b)[\ln|\sin(b+x)| - \ln|\sin(a+x)|] + C$
 (r) $-x + \cot a \ln|\cos x| - \cot a \ln|\cos(a+x)| + C$